$$\frac{\theta_m(s)}{V_a(s)} = \frac{K_a}{s(sR_a J_{eff} + R_a f_{eff} + K_a K_g)} = \frac{K}{s(T_m s + 1)}$$

$$K \triangleq \frac{K_a}{R_a f_{eff} + K_a K_g}$$

$$T_M \triangleq \frac{R_a J_{eff}}{R_a f_{eff} + K_a K_g}$$

$$\frac{\theta_L(s)}{V_a(s)} = \frac{nK}{s(T_m s + 1)}$$

<u>Positional Control for a Single Joint</u>

$$V_a(t) = \frac{K_p\, e(t)}{n} = \frac{K_p(\theta_{L\,desired} - \theta_{L\,actual})}{n} \qquad V_a(s) = \frac{K_p E(s)}{n}$$

$$\frac{\theta_L(s)}{E(s)} \triangleq G(s) = \frac{K_a K_p}{s(sR_a J_{eff} + R_a f_{eff} + K_a K_g)}$$

$$\frac{\theta_{L\,act}(s)}{\theta_{L\,des}(s)} \triangleq \frac{G(s)}{1+G(s)} = \frac{K_a K_p / R_a J_{eff}}{s^2 + [(R_a J_{eff} + K_a K_g)/R_a J_{eff}]s + K_a K_g/R_a J_{eff}}$$

In order to increase the system response time and reduce the steady-state error, increase the positional feedback gain and add some damping into the system by adding a derivative of the positional error.

$$V_a(t) = \frac{K_p[\theta_{L\,des} - \theta_{L\,actual}] + K_v[\dot{\theta}_{L\,des} - \dot{\theta}_{L\,act}]}{n} = \frac{K_p\, e(t) + K_v\, \dot{e}(t)}{n}$$

$$\frac{\theta_L(s)}{E(s)} \triangleq G_{PD}(s) = \frac{K_a K_v s + K_a K_p}{s(sR_a J_{eff} + R_a f_{eff} + K_a K_g)}$$

$$\frac{\theta_{L\,act}(s)}{\theta_{L\,des}(s)} \triangleq \frac{G_{PD}(s)}{1+G_{PD}(s)} = \frac{K_a K_v s + K_a K_p}{s^2 R_a J_{eff} + s(R_a f_{eff} + K_a K_g + K_a K_v) + K_a K_p}$$

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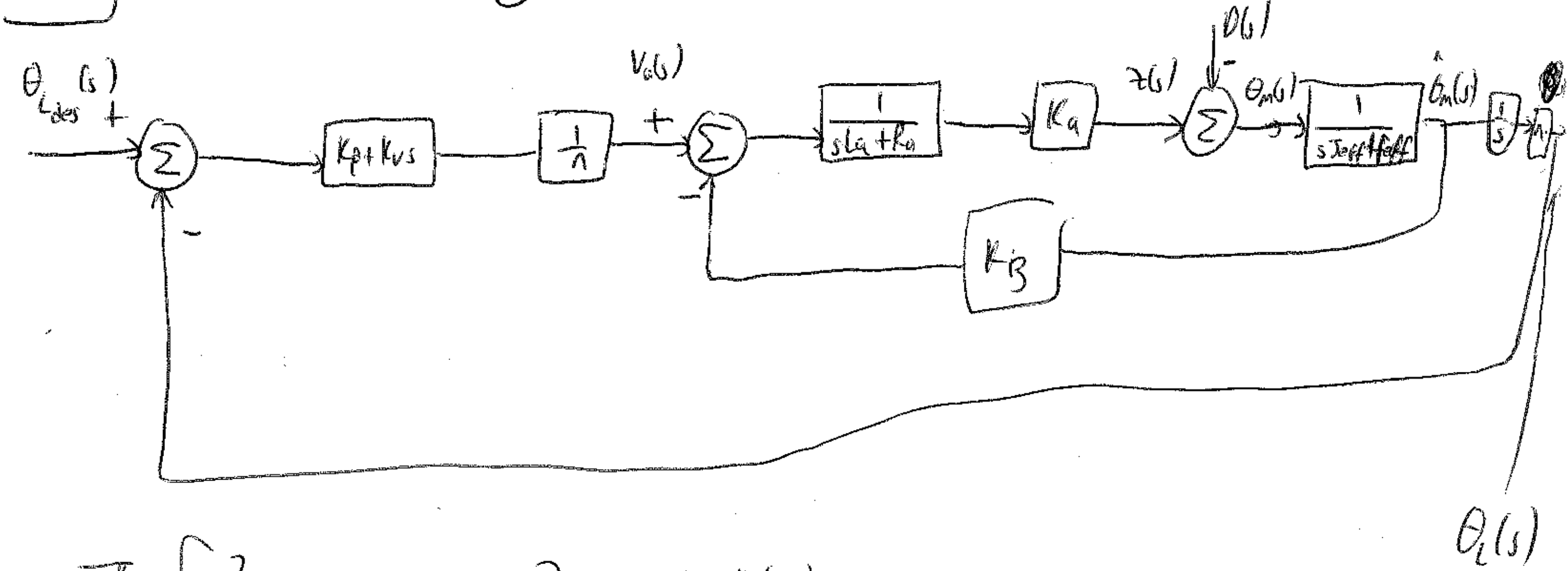

$$T=\left[s^2 J_{eff}+s f_{eff}\right]\Theta_m(s)+D(s)$$

$$\left.\frac{\theta_{L_{act}}(s)}{D(s)}\right|_{\theta_{L_{des}}(s)=0}=\frac{-nR_a}{s^2 R_a J_{eff}+s(R_a f_{eff}+K_a K_g+K_a K_v)+K_a K_p}$$

$$\theta_{L_{act}}(s)=\frac{K_a(K_p+K_v s)\theta_{L_{des}}(s)-nR_a D(s)}{s^2 R_a J_{eff}+s(R_a f_{eff}+K_a K_g+K_a K_v)+K_a K_p}$$

<u>Performance and stability criteria</u>

- fast rise time
- small or zero steady-state error
- fast settling time

$$s^2+2\xi\omega_n s+\omega_n^2=0 \qquad \omega_n^2=\frac{K_a K_p}{J_{eff}R_a} \qquad 2\xi\omega_n=\frac{R_a f_{eff}+K_a K_g+K_a K_v}{J_{eff}R_a}$$

For safety reasons the manipulator system cannot have an underdamped response for a step input

$$\xi\geq 1 \qquad K_p=\frac{\omega_n^2 J_{eff}R_a}{K_a}>0$$

Substituting ω_n

$$\xi=\frac{R_a f_{eff}+K_a K_g+K_a K_v}{2\sqrt{K_a K_p+J_{eff}R_a}}\geq 1 \qquad K_v\geq\frac{2\sqrt{K_a K_p J_{eff}R_a}-R_a f_{eff}-K_a K_g}{K_a}$$

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In order not to excite the structural oscillation and resonance of the joint, the undamped natural frequency w_n may be set to no more than 0,5 of the structural resonant frequency of the joint, that is

$$w_n \leq 0{,}5\, w_r$$

K_{stiff} : effective stiffness of the joint

stiffness: Direngenlik Sıkılık Sertlik

$Z_{restoring}$: $\boxed{K_{stiff}\, \theta_m(t) = Z_{inertia}}$ of the motor

$$J_{eff}\, \ddot{\theta}_m(t) + K_{stiff}\, \theta_m(t) = 0$$

$$J_{eff}\, s^2 + K_{stiff} = 0$$

$$w_r = \sqrt{\frac{K_{stiff}}{J_{eff}}} \qquad \text{is constant} \qquad w_0 = \sqrt{\frac{K_{stiff}}{J_0}}$$

$$\boxed{w_r = w_0 \sqrt{\frac{J_0}{J_{eff}}}}$$

$$w_n \leq 0{,}5\, w_r$$

$$0 < K_p < \frac{w_0^2\, J_0 R_a}{4 K_a} \qquad K_v \geq \frac{R_a w_0 \sqrt{J_0/J_{eff}} - R_a f_{eff} - K_a K_g}{K_a}$$

<u>Steady-State Errors of the system</u>

$$E(s) = \theta_{L des}(s) - \theta_{L act}(s)$$

i) For a step input of Magnitude A

$$\theta_{L des}(t) = A$$

$$e_{ss}(step) \triangleq e_{ssp} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} s\, \frac{\left[s^2 J_{eff} R_a + s(R_a f_{eff} + K_a K_g)\right]\frac{A}{s} + n R_a D(s)}{s^2 R_a J_{eff} + s(R_a f_{eff} + K_a K_g + K_a K_v) + K_a K_p}$$

Δ

$$= \lim_{s\to 0} s \left[\frac{(n R_a D(s))}{\Delta}\right]$$

$$Z_D(t) = Z_G(t) + Z_c(t) + Z_e$$

$$D(s) = \mathcal{T}_G(s) + \mathcal{T}_c(s) + \frac{\mathcal{T}_e}{s}$$

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$$E(s) = \frac{\Delta\, \theta_{Ldes}(s) + nR_a\left[\tau_G(s) + \tau_C(s) + \frac{\tau_e}{s} - \tau_{comp}(s)\right]}{\Delta}$$

$$e_{ssp} = \lim_{s \to 0} s \left[\frac{nR_a\left[\tau_G(s) + \tau_C(s) + \frac{\tau_e}{s} - \tau_{comp}(s)\right]}{\Delta}\right]$$

The steady-state position error due to the disturbance from centrifugal effect is zero as $t \to \infty$. The reason for that the centrifugal effect is proportional to $\dot{\theta}_L^2(t)$

If the disturbance is related to gravity loading of the link $\tau_{comp}(t)$ will be dominantly reflecting $\tau_G(t)$ and therefore

$$e_{ssp} = \frac{nR_a\tau_e}{K_aK_p}$$

recalling K_p bound

$$e_{ssp} = \frac{4n\tau_e}{w_0^2 J_0}$$ (very small since τ_e is very small)

(ii) For a ramp input of magnitude A

$\theta_{Ldes}(t) = At$

$\theta_{Ldes}(s) = \frac{A}{s^2}$

$D(s) = \tau_G(s) + \tau_C(s) + \frac{\tau_e}{s}$

$$e_{ss}(\text{ramp}) \triangleq e_{ssv} = \lim_{s \to 0} \frac{s\,[\Delta]\,\frac{A}{s^2} + nR_a\left[D(s) - \tau_{comp}(s)\right]}{\Delta}$$

$$= \underbrace{\frac{(R_a f_{eff} + K_aK_g)A}{K_aK_p}}_{\text{Term 1}} + \lim_{s \to 0} s\,\frac{nR_a\left[\tau_G(s) + \tau_C(s) + \frac{\tau_e(s)}{s} - \tau_{comp}(s)\right]}{\Delta}$$

$\tau_{comp}(t) \cong$ (gravity + centrifugal) torque demands

$e_{ss_v} = \text{Term 1} + e_{ssp}$

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$$\tau_i(t) = \sum_{k=i}^{6} \sum_{j=1}^{k} tr\left\{ \frac{\partial\, {}^0T_k}{\partial q_i} J_k \left(\frac{\partial\, {}^0T_k}{\partial q_j} \right)^T \right\} \ddot{q}_j(t) +$$

$$\sum_{r=i}^{6} \sum_{j=1}^{r} \sum_{k=1}^{r} tr\left\{ \frac{\partial^2\, {}^0T_r}{\partial q_j \partial q_k} J_r \left(\frac{\partial\, {}^0T_r}{\partial q_i} \right)^T \right\} \dot{q}_j(t)\, \dot{q}_k(t)$$

$$- \sum_{j=i}^{6} m_j\, g \left(\frac{\partial\, {}^0T_j}{\partial q_i} \right) \bar{r}_j \qquad \text{for } i = 1, 2, \ldots, 6$$

$$\tau_i(t) = \sum_{k=1}^{6} D_{ik}\, \ddot{q}_k(t) + \sum_{k=1}^{6} \sum_{m=1}^{6} h_{ikm}\, \dot{q}_k(t)\, \dot{q}_m(t) + C_i$$

$$\tau_i(t) = \begin{bmatrix} D_{i1} & D_{i2} & D_{i3} & D_{i4} & D_{i5} & D_{i6} \end{bmatrix} \begin{bmatrix} \ddot{q}_1(t) \\ \ddot{q}_2(t) \\ \ddot{q}_3(t) \\ \ddot{q}_4(t) \\ \ddot{q}_5(t) \\ \ddot{q}_6(t) \end{bmatrix} + \underbrace{\begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \dot{q}_3 & \dot{q}_4 & \dot{q}_5 & \dot{q}_6 \end{bmatrix} \begin{bmatrix} h_{i11} & h_{i12} & \cdots & h_{i16} \\ h_{i21} & h_{i22} & \cdots & \\ & & & \\ h_{i61} & & & h_{i66} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix}}_{\tau_i(t)}$$

$$+ C_i \rightarrow -\tau_G(t)$$

Computed Torque Control Technique

Controller for multi-joint Robots

$$\tau(t) = D_a(q_p) \left\{ \ddot{q}^d(t) + K_v \left[\dot{q}^d(t) - \dot{q}(t) \right] + K_p \left[q^d(t) - q(t) \right] \right\} + h_a(q, \dot{q}) + C_a(q)$$

LagrangeEuler description

K_v: 6x6 derivative

K_p: 6x6 position

feedback gain matrices

$$D(q)\ddot{q}(t) + h(q,\dot{q}) + C(q_1) = D_a(q_1)\left\{\ddot{q}_1^d(t) + K_v\left[\dot{q}^d(t) - \dot{q}(t)\right] + K_p\left[q^d(t) - q(t)\right]\right\}$$
$$+ h_a(q,\dot{q}) + C(q)$$

if $D_a(q)$, $h_a(q,\dot{q})$ and $C(q)$ are equal to $D(q)$, $h(q,\dot{q})$ and $C(q)$ then the above torque equality reduces to

$$D q\left(\ddot{e}(t) + K_v \dot{e}(t) + K_p e(t)\right] = 0 \qquad \text{where } e(t) = q^d(t) - q(t)$$

$D(q)$ is always non-singular. K_p and K_v can be chosen appropriately so the characteristic roots of this equation have negative real parts, then $e(t)$ asymptotically approaches to zero.

b

Lagrange Euler representation becomes inefficient in the computation of joint torques. Paul suggests that one must not design a closed-loop digital controller using L.E (Lagrange-Euler) representation such as neglecting the velocity related terms of $h_a(q,\dot{q})$ and the off-diagonal elements of the acceleration related matrix $D_a(q)$

Then

$$\tau(t) = \text{diag}\left[D_a(q)\right]\left\{q^d(t) + K_v\left[\dot{q}^d(t) - \dot{q}(t)\right] + K_p\left[q^d(t) - q(t)\right]\right\} + C_a(q)$$

Computer simulations conducted on this control law shows that terms cannot be neglected when the robot arm is moving at high speeds, 587 (1) 25-12-2002

Force control, compliness, path trajectory

örnek:
iki robotun
eş zamanlı
birşeyleri tutması

An analogous control law in the joint variable space can be derived from N-E equations of motion to serve a robot arm, The control law is computed recursively using N-E equations. The recursive control law can be obtained by substituting $\ddot{q}_i(t)$ into N-E equations to obtain the necessary joint torque for each actuator.

$$\ddot{q}_i(t) = \ddot{q}_i^{d}(t) + \sum_{j=1}^{n} K_v^{ij}\left[\dot{q}_j^{d}(t) - \dot{q}_j(t)\right] + \sum_{j=1}^{n} K_p^{ij}\left[q_j^{d}(t) - q_j(t)\right]$$

1. The first term generates desired torque for each joint if there is no modelling error and the physical system parameters are known, However there are errors due to backlash gear friction uncertainity about the inertia parameters and time delay into the servo loop so that deviation from the desired joint trajectory will be inevitable.

2. The remaining terms in N.E eqns of motion will generate the correction torques, to compansate for small deviations, from the desired ~~trajectory~~ joint trajectory,

Motion Controls

1. Joint Motion Control
   - 1.1 Joint servo mechanism
   - 1.2 computed torque technique
   - 1.3 Minimum time control
   - 1.4 Variable structure control
   - 1.5 Nonlinear decoupled control
2. Resolved Motion Controls
   - 2.1 Resolved motion rate control
   - 2.2 Resolved " acceleration control

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2.3 Resolved motion force control

3. Adaptive Control
   3.1 Model Referenced adaptive control
   3.2 Self-Tuning adaptive control
   3.3 Adaptive perturbation control with feedforward compensation
   3.4 Resolved motion adaptive control