26-12-2015

MECE 401 Final

Name:
Surname:

Q1) a) Draw the state flow diagram of the robot joint angular velocity control system with the following equations:

$$w_{ref}(s) - w(s) = e(s)$$
$$e(s)C(s) = V_a(s)$$
$$w(s)K_b = e_a(s)$$
$$\frac{V_a(s) - e_a(s)}{sL + R} = I(s)$$
$$I(s)K_a = T(s)$$
$$\frac{T(s) - D(s)}{sJ + B} = w(s)$$

In these equations $w_{ref}(s)$ is the reference angular velocity value, w(s) is the actual angular velocity value (output), e(s) is the error function, C(s) is the controller transfer function, $V_a(s)$ applied voltage to motor, K_b is the motor speed constant, $e_a(s)$ is the produced back-electromotive force at the motor, L is the inductance value of the armature, R is the resistance value of the armature, I(s) is the armature current, K_a is the motor torque constant, D(s) is the disturbance, J is the motor moment of inertia, B ,s the motor viscous friction coefficient. **(20 points)**

b) The system parameters are: L=0.1 Henry, R=1 Ohm, K_b=1 Volt.sec/rad, K_a= 1Newton.m/Ampere, J=1 kg.m^2, B=1 Newton.m.sec.Assuming that there is no disturbance (D(s)=0), find $\frac{e(s)}{w_{ref}(s)}$. **(10 points)**

c) If C(s)=K_p where K_p=5 (a proportional 'P' controller) and the reference signal is given as a unit step reference change of 10 rad/sec (meaning $w_{ref}(s) = \frac{10}{s}$) what will be the steady state error e_{ss}. **(10 points)**

d) If $C(s) = \frac{K_I}{s}$ where K_I=5 (an integral 'I' controller) and the reference signal is given as a unit step reference change of 10 rad/sec (meaning $w_{ref}(s) = \frac{10}{s}$) what will be the steady state error e_{ss}. **(10 points)**

Q2) The transformation matrixes of a two-link manipulator where the first joint is a revolute joint and the second joint is a prismatic joint is given by the formulas

$$A_1 = \begin{bmatrix} Cos(\theta_1) & -Sin(\theta_1) & 0 & \lambda_1 Cos(\theta_1) \\ Sin(\theta_1) & Cos(\theta_1) & 0 & \lambda_1 Sin(\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where θ_1 is the joint variable (angle) for joint 1, λ_1 is the length of link1 and d_2 is the joint variable for joint 2 (distance).

a) Draw the structure (geometry) of the two-link manipulator. In your illustration also indicate the coordinate frames of the corresponding joints and the joint variables and parameters (θ_1, λ_1 and d_2). **(20 points)**

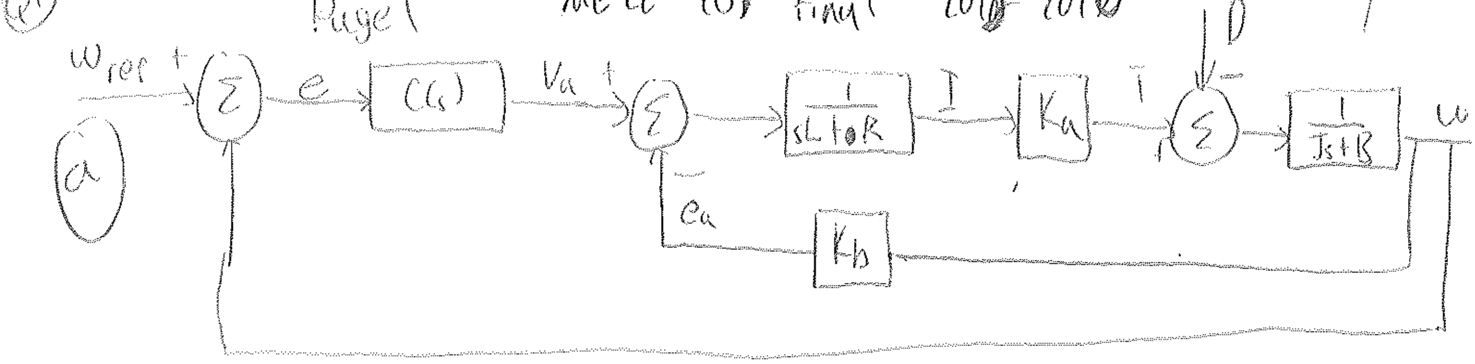
b) The mass of link 1 (should be confined over λ_1) connected to the joint 1 is m_1 and the mass of the prismatic joint (should be confined over d_2) is m_2. These masses are evenly distributed over link 1 and the prismatic joint. Hence the center of mass of the link 1 and the prismatic joint are at the mid points of link1 and prismatic joint respectively. Find the total kinetic energy and the total potential energy of this manipulator. **(20 points).**

Hint: While finding the Kinetic energies, be careful about the second joint. The center of mass of second joint has linear velocities in x, y and z directions.

Q3) The Lagrangian function of a two link manipulator is given by the function $L = A_1\dot{\theta}_1^2 + A_2\dot{d}_2^2 + A_3\sin(\theta_1)$ where θ_1 (angle) and d_2 (distance) are the joint variables for joint 1 and joint 2 respectively. Find the corresponding equivalent torque τ_1 and the force F_1 acting on joint 1 and joint 2 respectively.

Q1 Page 1 MECE 401 Final 2015-2016

(a)



$$\left.\frac{w(s)}{V_a(s)}\right| = \frac{\frac{1}{sL+R} K_a \frac{1}{Js+R}}{1+\frac{1}{sL+R}K_a\frac{1}{Js+R}K_b} = \frac{K_a}{[sL+R][Js+B]+K_aK_b} = H(s)$$

(b)

$$\left.\frac{w(s)}{w_{ref}(s)}\right|_{D(s)=0} = \frac{C(s)H(s)}{1+C(s)H(s)} = \frac{w_{ref}(s)-e}{w_{ref}(s)} \qquad \frac{e(s)}{w_{ref}(s)} = \frac{1}{C(s)H(s)+1}$$

$D(s)=0$

$$\frac{e(s)}{w_{ref}(s)} = \frac{1}{C(s)H(s)+1} = \frac{1}{\frac{K_a}{[sL+R][Js+B]+K_aK_b}C(s)+1}$$ (1)

(c) $L=0.1H$ $R=1\Omega$ $K_b=1\ \frac{Volt.s}{rad}$ $K_a=1\ \frac{Newton.m}{Ampere}$ $B=1\ \frac{Newton.m.s}{sec}$

$J=1\ kgm^2$ $C(s)=K_p=50$ $w_{ref}(s)=\frac{10}{s}$

$$\frac{e(s)}{w_{ref}(s)} = \frac{1}{\frac{1}{[0.1s+1][s+1]+1}K_p+1} = \frac{[0.1s+1][s+1]+1}{K_p+[0.1s+1][s+1]+1}$$

$$e(s) = \frac{[0.1s+1][s+1]+1}{[0.1s+1][s+1]+1+K_p}w_{ref}(s)$$

$$\lim_{s\to0} s\,e(s) = \lim_{s\to0} s\frac{10}{s}\left[\frac{[0.1s+1][s+1]+1}{[0.1s+1][s+1]+1+K_p}\right] = \lim_{s\to0} 10\frac{1}{1+1+50} = \frac{10}{50} = 0.2$$

Put the parameters

(b) $$\frac{e(s)}{w_{ref}(s)} = \frac{1}{\frac{1}{[0.1s+1][s+1]+1}C(s)+1} = \frac{[0.1s+1][s+1]+1}{[0.1s+1][s+1]+1+C(s)}$$

(c) $C(s) = K_p = 5$ $\quad w_{ref}(s) = \frac{10}{s}$

$$\frac{e(s)}{w_{ref}(s)} = \frac{[0.1s+1][s+1]+1}{[0.1s+1][s+1]+1+5}$$

$$e(s) = \frac{10}{s}\,\frac{[0.1s+1][s+1]+1}{[0.1s+1][s+1]+1+5}$$

$$\lim_{s\to 0} s\,e(s) = \lim_{t\to\infty} e(t) = \lim_{s\to 0} \cancel{s}\,\frac{10}{\cancel{s}}\,\frac{[0.1s+1][s+1]+1}{[0.1s+1][s+1]+1+5} = \frac{10\,[1+1]}{7} = \frac{20}{7}$$

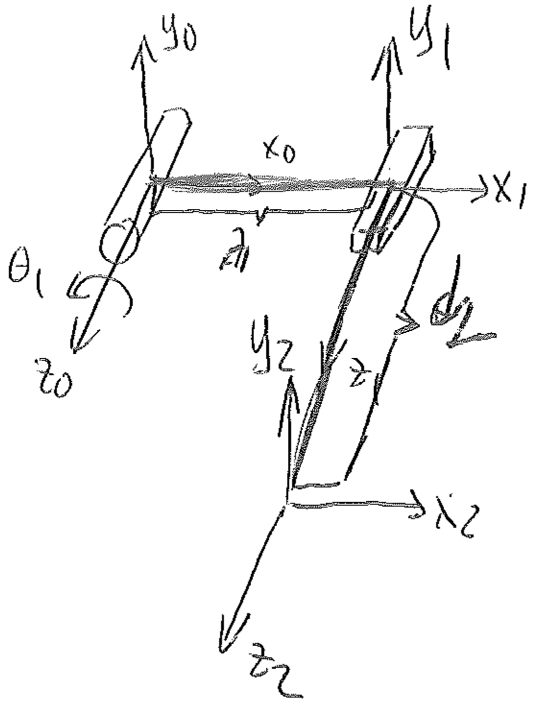
(d) $C(s) = \frac{K_I}{s} = \frac{5}{s}$ $\quad w_{ref}(s) = \frac{10}{s}$

$$\frac{e(s)}{w_{ref}(s)} = \frac{[0.1s+1][s+1]+1}{[0.1s+1][s+1]+1+\frac{5}{s}}$$

$$e(s) = \frac{10}{s}\,\frac{s[0.1s+1][s+1]+s}{s[0.1s+1][s+1]+s+5}$$

$$\lim_{s\to 0} s\,e(s) = \lim_{t\to\infty} e(t) = \lim_{s\to 0} \cancel{s}\,\frac{10}{\cancel{s}}\,\frac{s[0.1s+1][s+1]+s}{s[0.1s+1][s+1]+s+5} = \frac{0}{5} = 0$$

Q2 (a)



Manipulator Geometry

Q2

(b) center of mass of m_1 $\rightarrow$ $z_1 = 0$, $x_1 = \frac{a_1}{2}\cos\theta_1$, $y_1 = \frac{a_1}{2}\sin\theta_1$

$$\dot{x}_1 = -\frac{a_1}{2}[\sin\theta_1]\dot{\theta}_1 \qquad \dot{y}_1 = \frac{a_1}{2}[\cos\theta_1]\dot{\theta}_1 \qquad \dot{z}_1 = 0$$

linear speed of center of mass m_1

$$v_1 = \sqrt{\dot{x}_1^2 + \dot{y}_1^2 + \dot{z}_1^2} = \sqrt{\frac{a_1^2}{4}\dot{\theta}_1^2[\sin^2\theta_1 + \cos^2\theta_1]} = \frac{a_1}{2}\dot{\theta}_1$$

kinetic energy of m_1

$$K_1 = \frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_1\left[\frac{a_1^2}{4}\dot{\theta}_1^2\right]$$

(b)

center of mass of m_2 $\rightarrow$ $x_2 = a_1\cos\theta_1$, $y_2 = a_1\sin\theta_1$, $z_2 = \frac{d_2}{2}$

$$\dot{x}_2 = -a_1[\sin\theta_1]\dot{\theta}_1$$

$$\dot{y}_2 = a_1[\cos\theta_1]\dot{\theta}_1$$

$$\dot{z}_2 = \frac{\dot{d}_2}{2}$$

linear speed of center of mass m_2

$$v_2 = \sqrt{\dot{x}_2^2 + \dot{y}_2^2 + \dot{z}_2^2} = \sqrt{a_1^2\dot{\theta}_1^2[\sin^2\theta_1 + \cos^2\theta_1] + \frac{\dot{d}_2^2}{4}}$$

$$= \sqrt{a_1^2\dot{\theta}_1^2 + \frac{\dot{d}_2^2}{4}}$$

Kinetic energy of m_2

$$K_2 = \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_2\left[a_1^2\dot{\theta}_1^2 + \frac{\dot{d}_2^2}{4}\right]$$

(Q2 b) continue

$$K = K_1 + K_2 = \frac{1}{8} m_1 a_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 \left[a_1^2 \dot{\theta}_1^2 + \frac{\dot{d}_2^2}{4} \right]$$

$$P_1 = m_1 g y_1 = m_1 g \frac{a_1}{2} \sin\theta_1 \qquad P_2 = m_2 g y_2 = m_2 g a \sin\theta_1$$

(Q3) $L = A_1 \dot{\theta}_1^2 + A_2 \dot{d}_2^2 + A_3 \sin(\theta_1)$

$$\tau_1 = \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_1} - \frac{\partial L}{\partial \theta_1} \qquad F_2 = \frac{d}{dt} \frac{\partial L}{\partial \dot{d}_2} - \frac{\partial L}{\partial d_2}$$

$$\frac{\partial L}{\partial \theta_1} = A_3 \cos(\theta_1) + 0 \qquad \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_1} = 2A_1 \ddot{\theta}_1$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = 2A_1 \dot{\theta}_1 + 0 \qquad \boxed{\tau_1 = 2A_1 \ddot{\theta}_1 - A_3 \cos(\theta_1)}$$

Torque of first joint

$$\frac{\partial L}{\partial d_2} = 0 \qquad \frac{d}{dt} \frac{\partial L}{\partial \dot{d}_2} = 2A_2 \ddot{d}_2$$

$$\frac{\partial L}{\partial \dot{d}_2} = 2A_2 \dot{d}_2 \qquad \boxed{F_1 = 2A_2 \ddot{d}_2}$$

Force of second joint