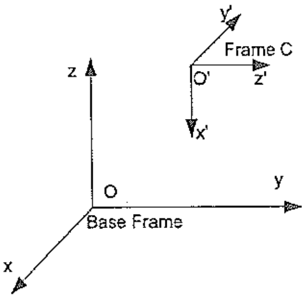
MECE401

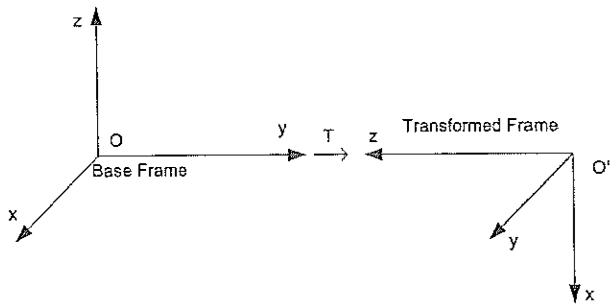
MIDTERM

Date: 09-12-2014

Q1) The frame C(x',y',z') as shown in the figure below and its origin is at O'(1,5,9) with respect to the base frame. If point P(1,-2,1) is defined in the coordinate frame C, find its coordinates at the base frame. **(20 points)**



Q2) Let a transformation T be defined as shown in the figure below where origins of base and transformed frame are O and O' and they are the same points. Define the transformation T in terms of Euler angles φ, θ, ψ. **(20 points)**



Q3) Let the differential translation and rotation vectors be defined as

$$^{i-1}\underline{d} = i + 0.5j + 0k$$

$$^{i-1}\underline{\delta} = 0.1i + 0j + 0k$$

with reference to (i-1)th link coordinate frame of a manipulator. If $^{i-1}A_i$ is the transformation between (i-1)th link and ith link coordinate frames find the differential translation and rotation vectors with reference to ith coordinate frame. **(20 points)**

Q4) Consider a manipulator with three degree of freedom where the first two joint variables are θ_1 and θ_2 (revolute joints) and the third joint variable is d_3 (prismatic joint).
If a joint is prismatic the transformation matrix A_i is of the form

$$A_i = Rot(z_{i-1},\theta_i)\times Transl(0,0,d_i)\times Rot(x_i,\alpha_i) = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & 0 \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & 0 \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If a joint is revolute the transformation matrix A_i is of the form
$A_i = Rot(z_{i-1},\theta_i)\times Transl(0,0,d_i)\times Transl(\lambda_i,0,0)\times Rot(x_i,\alpha_i)$

$$A_i = \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & \lambda_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\alpha_i) & -\cos(\theta_i)\sin(\alpha_i) & \lambda_i\sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The table where these joint parameters and variables are tabulated for these three joints is as below:

Link i	d_i *(meter)*	λ_i *(meter)*	α_i *(degree)*	θ_i *(degree)*
1	*0*	*1*	*-90*	θ_1 *(joint variable)*
2	*0*	*1*	*90*	θ_2 *(joint variable)*
3	d_3 *(joint variable)*	*0*	*0*	*-90*

a) Find A_1, A_2 and A_3. **(20 points)**

b) Draw the manipulator structure, assign the link coordinate frames and show all these joint parameters and variables over the manipulator structure. **(20 points)**

Q1

$$C = \begin{array}{c} \begin{array}{cccc} x' & y' & z' & \end{array} \\ \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 5 \\ -1 & 0 & 0 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

let $P = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad P' = CP = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 5 \\ -1 & 0 & 0 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 9 \\ 1 \end{bmatrix}$ //

let $P = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ $\quad P' = CP = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 5 \\ -1 & 0 & 0 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 8 \\ 1 \end{bmatrix}$ //

let $P = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ $\quad P' = CP = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 5 \\ -1 & 0 & 0 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 9 \\ 1 \end{bmatrix}$ //

let $P = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ $\quad P' = CP = \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 5 \\ -1 & 0 & 0 & 9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 9 \\ 1 \end{bmatrix}$ //

Q2

$$\begin{bmatrix} \cos\phi\cos\theta\cos\psi - \sin\phi\sin\psi & -\cos\phi\cos\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\sin\theta & 0 \\ \sin\phi\cos\theta\cos\psi + \cos\phi\sin\psi & -\sin\phi\cos\theta\sin\psi - \cos\phi\cos\psi & \sin\phi\sin\theta & 0 \\ -\sin\theta\cos\psi & \sin\theta\sin\psi & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \boxed{1} & \boxed{0} & 0 \\ 0 & 0 & \boxed{-1} & 0 \\ -1 & 0 & \boxed{0} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\cos\theta = 0 \longrightarrow \theta = 90, -90$

let $\theta = 90$ // $\longrightarrow \sin\theta = 1$, $\cos\theta = 0$

$\underbrace{\sin\theta}_{1}\cos\phi = 0 \Rightarrow \cos\phi = 0$

$\underbrace{\sin\theta}_{1}\sin\phi = -1 \Rightarrow \sin\phi = -1$

$\rightarrow \phi = -90^\circ$ //

$-\underbrace{\cos\phi}_{0}\cos\theta\sin\psi - \underbrace{\sin\phi}_{-1}\cos\psi = 1 \Rightarrow \cos\psi = 1 \quad \psi = 0$ //

(Q3)

$$\begin{array}{ccc} & {}^{i-1}A_i & \\ \bullet & \longrightarrow & \bullet \\ {}^{i-1}\Delta \downarrow & & \downarrow {}^{i}\Delta \\ \bullet & \longrightarrow & \bullet \\ & {}^{i-1}A_i & \end{array}$$

$${}^{i-1}\Delta \; {}^{i-1}A_i = {}^{i-1}A_i \; {}^{i}\Delta$$

$${}^{i-1}d = i + 0.5j + 0k,$$

$${}^{i-1}\delta = 0.1i + 0j + 0k$$

$${}^{i-1}A_i = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{i}\delta_x = {}^{i-1}\delta \cdot n = 0.1 n_x$$

$${}^{i}\delta_y = {}^{i-1}\delta \cdot o = 0.1 o_x$$

$${}^{i}\delta_z = {}^{i-1}\delta \cdot a \quad 0.1 a_x$$

$${}^{i}d_x = {}^{i-1}\delta \cdot (p \times n) + d \cdot n$$

$${}^{i-1}\delta \times p = \begin{vmatrix} i & j & k \\ 0.1 & 0 & 0 \\ p_x & p_y & p_z \end{vmatrix}$$

$${}^{i}d_x = n \cdot (({}^{i-1}\delta \times p) + d)$$

$${}^{i}d_y = o \cdot (({}^{i-1}\delta \times p) + d)$$

$${}^{i}d_z = a \cdot (({}^{i-1}\delta \times p) + d)$$

$${}^{i-1}\delta \times p = [\, i \times 0 \; j(0.1p_z) + k\, 0.1p_y \,]$$

$${}^{i-1}\delta \times p = [\, 0i - 0.1p_z j + 0.1p_y k \,]$$

$$({}^{i-1}\delta \times p) + d = 0.1i - 0.1p_z j + 0.1p_y k$$

$${}^{i}d_x = 0.1n_x - 0.1p_z n_y + 0.1p_y n_z$$

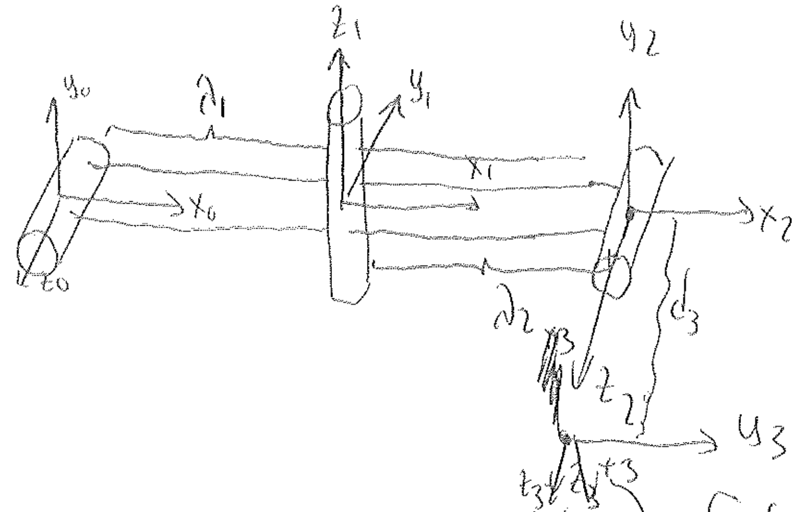
$${}^{i}d_y = 0.1o_x - 0.1p_z o_y + 0.1p_y o_z$$

$${}^{i}d_z = 0.1a_x - 0.1p_z a_y + 0.1p_y a_z$$

$${}^{i}\Delta = \begin{bmatrix} 0 & 0.1a_x & 0.1o_x & {}^{i}d_x \\ 0.1a_x & 0 & 0.1n_x & {}^{i}d_y \\ 0.1o_x & 0.1n_x & 0 & {}^{i}d_z \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Q-4

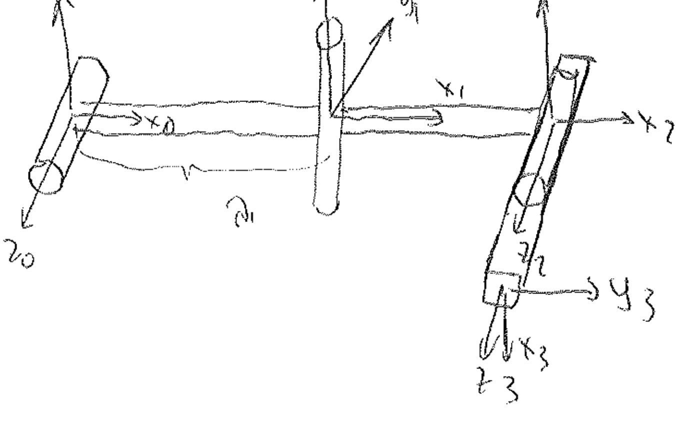
(b)



(a) $$A_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \times 0 & -\sin\theta_1 & \cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 \times 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & \cos\theta_1 \\ \sin\theta_1 & 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & \cos\theta_2 \\ \sin\theta_2 & 0 & -\cos\theta_2 & \sin\theta_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



MECE401 Final 2014 2015

Q1) The lagrangian L=K-P where K stands for kinetic energy and P stands for the potential energy for a two-link manipulator is given as

$$L = a\dot{\theta}_1^2 + b\dot{\theta}_1\dot{\theta}_2 + c\dot{\theta}_2^2 + d(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)Cos(\theta_2) + eCos(\theta_1) + fCos(\theta_1 + \theta_2)$$

Find the expression for the joint torques at joints 1 and 2.

Hint: Use the formula below for the joint torques where τ_i stands for the joint torque for the joint i.

$$\tau_i = \frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_i} - \frac{\partial L}{\partial \theta_i}$$

Q2) Given the force and moment applied in base coordinates as;

$$f = \begin{bmatrix} -5 & 4 & 0 \end{bmatrix}$$

and

$$m = \begin{bmatrix} 0 & -200 & 50 \end{bmatrix}$$

determine the equivalent force ^{T}f and moment ^{T}m for the frame T.

$$T = \begin{bmatrix} 1 & 0 & 0 & -10 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(Q1)

$$\frac{\partial L}{\partial \dot{\theta}_1} = 2a\dot{\theta}_1 + b\dot{\theta}_2 + 2d\dot{\theta}_1\cos\theta_2 + d\dot{\theta}_2\cos\theta_2$$

$$\frac{\partial}{dt}\frac{\partial L}{\partial \dot{\theta}_1} = 2a\ddot{\theta}_1 + b\ddot{\theta}_2 + 2d\ddot{\theta}_1\cos\theta_2 + 2d\dot{\theta}_1\dot{\theta}_2(-\sin\theta_2)$$
$$+ d\ddot{\theta}_2\cos\theta_2 + d\dot{\theta}_2^2(-\sin\theta_2)$$

$$= 2a\ddot{\theta}_1 + b\ddot{\theta}_2 + 2d\ddot{\theta}_1\cos\theta_2 - 2d\dot{\theta}_1\dot{\theta}_2\sin\theta_2$$
$$+ d\ddot{\theta}_2\cos\theta_2 - d\dot{\theta}_2\sin\theta_2$$

$$\frac{\partial L}{\partial \theta_1} = -e\sin\theta_1 - f\sin(\theta_1+\theta_2)$$

$$T_1 = 2a\ddot{\theta}_1 + b\ddot{\theta}_2 + 2d\ddot{\theta}_1\cos\theta_2 - 2d\dot{\theta}_1\dot{\theta}_2\sin\theta_2 + d\ddot{\theta}_2\cos\theta_2$$
$$- d\dot{\theta}_2\sin\theta_2 + e\sin\theta_1 + f\sin(\theta_1+\theta_2)$$

$$\frac{\partial L}{\partial \dot{\theta}_2} = b\dot{\theta}_1 + d\dot{\theta}_1\cos\theta_2 + 2c\dot{\theta}_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = b\ddot{\theta}_1 + d\ddot{\theta}_1\cos\theta_2 + d\dot{\theta}_1\dot{\theta}_2(-\sin\theta_2) + 2c\ddot{\theta}_2$$

$$\frac{\partial L}{\partial \theta_2} = -d(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)\sin(\theta_2) - f\sin(\theta_1+\theta_2)$$

$$T_2 = b\ddot{\theta}_1 + 2c\ddot{\theta}_2 + d\ddot{\theta}_1\cos\theta_2 - d\dot{\theta}_1\dot{\theta}_2\sin\theta_2$$
$$+ d(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)\sin\theta_2 + f\sin(\theta_1+\theta_2)$$

Correct

for joint 2

$$\frac{\partial L}{\partial \dot{\theta}_2} = b\dot{\theta}_1 + 2c\dot{\theta}_2 + d\dot{\theta}_1 \cos\theta_2$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = b\ddot{\theta}_1 + 2c\ddot{\theta}_2 + d\ddot{\theta}_1 \cos\theta_2 - d\dot{\theta}_1\dot{\theta}_2 \sin\theta_2$$

$$\frac{\partial L}{\partial \theta_2} = -d(\dot{\theta}_1^2 + \dot{\theta}_1\dot{\theta}_2)\sin\theta_2 \quad - f\sin(\theta_1+\theta_2)$$

$$\tau_2 = \ddot{\theta}_1(b + d\cos\theta_2) + 2c\ddot{\theta}_2 + d\dot{\theta}_1^2 \sin\theta_2 + f\sin(\theta_1+\theta_2) \quad \left(\text{correct same}\right)$$

2) 50

$$(f\times p)+m = \left\{ \underbrace{\begin{vmatrix} i & j & k \\ -5 & 4 & 0 \\ -10 & 2 & 0 \end{vmatrix}}_{\substack{= k(-10+40) \\ = 30k}} + 0i - 200j + 80k \right\} = 0i - 200j + 80k$$

$${}^{T}m_x = [1 \ \ 0 \ \ 0]\begin{bmatrix} 0 \\ -200 \\ 80 \end{bmatrix} = 0$$

$${}^{T}m_y = [0 \ \ 0 \ \ -1]\begin{bmatrix} 0 \\ -200 \\ 80 \end{bmatrix} = -80$$

$${}^{T}m_z = [0 \ \ 1 \ \ 0]\begin{bmatrix} 0 \\ -200 \\ 80 \end{bmatrix} = -200$$

$$\underline{{}^{T}m = 0i - 80j - 200k}$$

$${}^{T}f_x = n.f = [1 \ 0 \ 0]\begin{bmatrix} -5 \\ 4 \\ 0 \end{bmatrix} = -5$$

$${}^{T}f_y = o.f = [0 \ \ 0 \ \ -1]\begin{bmatrix} -5 \\ 4 \\ 0 \end{bmatrix} = 0$$

$${}^{T}f_z = a.f = [0 \ \ 1 \ \ 0]\begin{bmatrix} -5 \\ 4 \\ 0 \end{bmatrix} = 4$$

$$\underline{{}^{T}f = -5i + 0j + 4k}$$

10-11-2015

MECE401 Midterm

Name:
Surname:
Number:

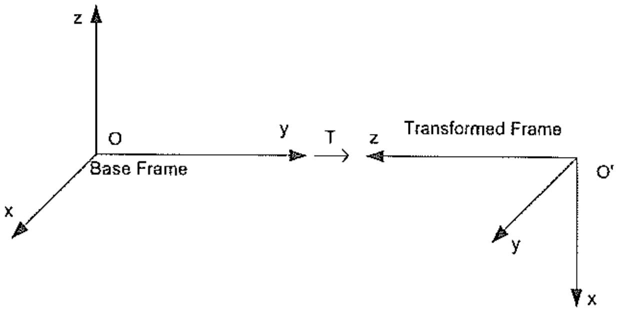
Q1) A transformation M with respect to the base coordinate frame is given by:

$$M = \begin{bmatrix} 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a) Draw the base coordinate frame and transformed coordinate frame. **(25 points)**

b) If the origin of the base coordinate frame is $o = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$ find the origin of the transformed coordinate frame o' with respect to the base coordinate frame. **(5 points)**

Q2) Let a transformation T be defined as shown in the figure below where origins of base and transformed frame are O and O' and they are at the same points. Define the transformation T in terms of roll, pitch and yow angles φ, θ, ψ. **(30 points)**



Q3) A rotational transformation of θ degrees around an axis of rotation k is given by,

$$T = \begin{bmatrix} \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find the axis of rotation k and the angle of rotation θ. **(40 points)**

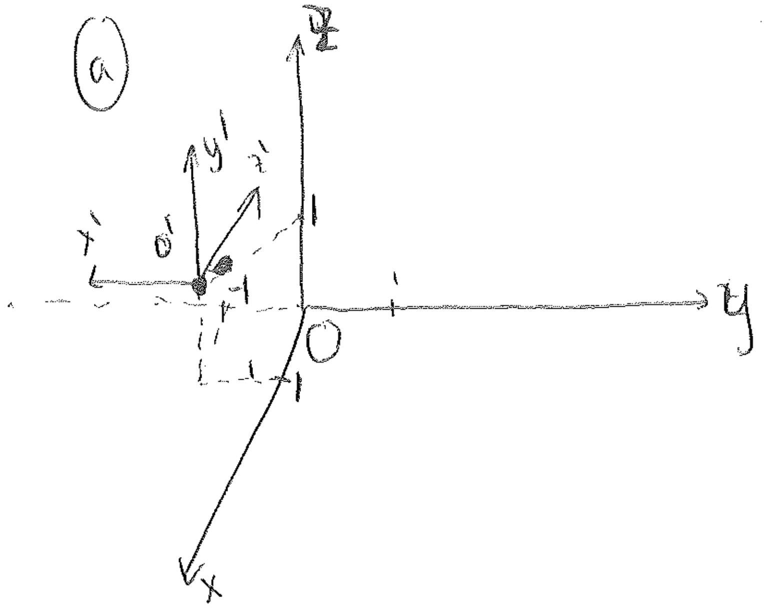
Q1

2015-2016 Midterm

MECE 401

$$M = \begin{array}{c} \begin{matrix} x' & y' & z' & \end{matrix} \\ \begin{bmatrix} 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(a)



$x' \to$ in $-y$ direction

$y' \to$ in z direction

$z' \to$ in $-x$ direction

(b) $^{base}o' = M\,{}^{M}o$

$$= \begin{bmatrix} 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix} = {}^{base}o'$$

(Q2)

$$T = \begin{array}{c} \begin{array}{cccc} x' & y' & z' & \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

15 point

Transformed axis

x' in $-z$ direction

y' in x direction

z' in $-y$ direction

$$T = RPY(\phi, \theta, \psi) = \begin{bmatrix} c(\phi)c(\theta) & c(\phi)s(\theta)s(\psi) - s(\phi)c(\psi) & c(\phi)s(\theta)c(\psi) + s(\phi)s(\psi) & 0 \\ s(\phi)c(\theta) & s(\phi)s(\theta)s(\psi) + c(\phi)c(\psi) & s(\phi)s(\theta)c(\psi) - c(\phi)s(\psi) & 0 \\ -s(\theta) & c(\theta)s(\psi) & c(\theta)c(\psi) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$s(\theta) = \sin\theta$, $s(\psi) = \sin(\psi)$

$c(\theta) = \cos\theta$, $c(\psi) = \cos(\psi)$

$s(\phi) = \sin\phi$

$c(\phi) = \cos\phi$

index (3,1) 5 point

$-\sin\theta = -1 \Rightarrow \sin\theta = 1 \Rightarrow \underline{\theta = 90}$ 5 point

if $\theta = 90 \rightarrow \cos\theta = 0$, $\rightarrow \sin\theta = 1$

if $\sin\theta = 1$

15 points

index (1,2) $\cos(\phi)\underbrace{\sin(\theta)}_{1}\sin(\psi) - \sin(\phi)\cos(\psi) = 1$

$\sin(\psi - \phi) = 1$

$\psi - \phi = 90^\circ$

index (1,3) $\cos(\phi)\underbrace{\sin(\theta)}_{1}\cos(\psi) + \sin(\phi)\sin(\psi) = 0$

$\cos(\psi - \phi) = 0$

$\psi - \phi = 90^\circ$

(same result)

(Q3)

$$T = \begin{bmatrix} \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} n_x & o_x & a_x & 0 \\ n_y & o_y & a_y & 0 \\ n_z & o_z & a_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 1

$$\cos\theta = \frac{1}{2}\left[n_x + o_y + a_z - 1\right] = \frac{1}{2}\left[\frac{1}{2} + 0 + \frac{1}{2} - 1\right] = 0 \qquad \cos\theta = 0$$

$$\theta = 90^0$$

$$\sin\theta = +\frac{1}{2}\left[(o_z - a_y)^2 + (a_x - n_z)^2 + (n_y - o_x)^2\right]^{\frac{1}{2}}$$

$$= \frac{1}{2}\left[\left(\frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)\right)^2 + \left(\frac{1}{2} - \frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)\right)^2\right]^{\frac{1}{2}}$$

$$\sin\theta = \frac{1}{2}\left[2 + 0 + 2\right]^{\frac{1}{2}} = 1 \qquad \sin\theta = 1 \quad \theta = 90^0$$

Step 2

$$(o_z - a_y) = \frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right) = \sqrt{2} \qquad (a_x - n_z) = \frac{1}{2} - \frac{1}{2} = 0 \qquad (n_y - o_x) = \left(\frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)\right) = \sqrt{2}$$

largest

$$k_x = \mathrm{Sgn}\,(o_z - a_y)\sqrt{\frac{n_x - \cos\theta}{1 - \cos\theta}} = \mathrm{Sgn}(\sqrt{2}) \times \sqrt{\frac{\frac{1}{2} - 0}{1}} = \frac{1}{\sqrt{2}}$$

$$k_y = \frac{n_y + o_x}{2k_x \,\mathrm{Vers}\,\theta} = \frac{0}{2 \times \frac{1}{\sqrt{2}} \times 1} = 0 \qquad k_z = \frac{n_z + a_x}{2k_x \,\mathrm{Vers}\,\theta} = \frac{1}{2 \times \frac{1}{\sqrt{2}} \times 1} = \frac{1}{\sqrt{2}}$$