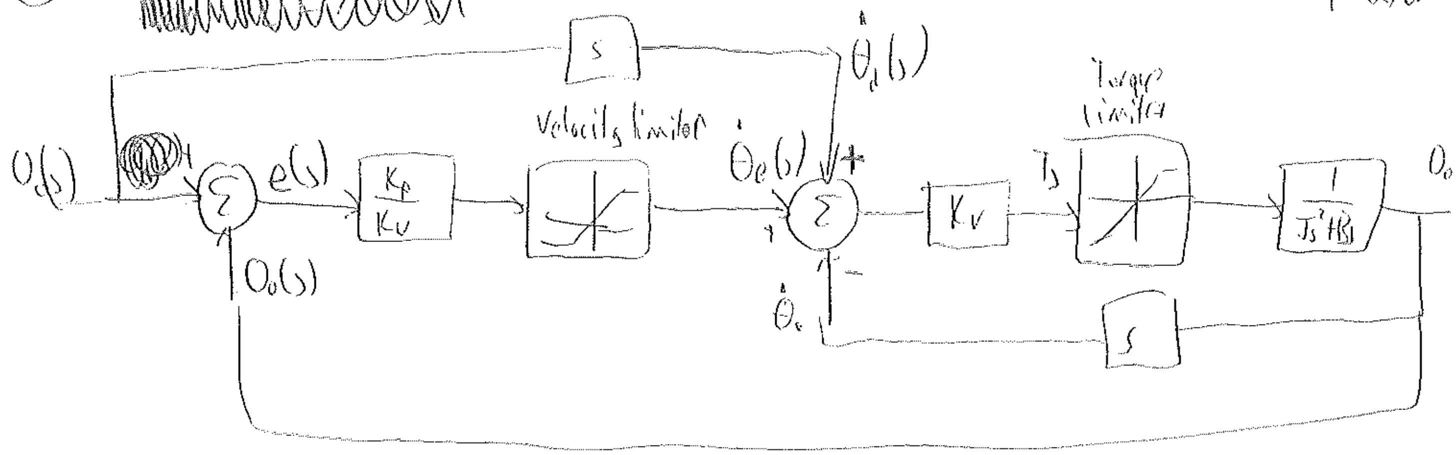
(Q1) Controller for controlling the slowest joint of a manipulator



(i) $H(s) = \frac{\theta_o(s)}{\theta_d(s)} = ?$ (Find transfer function)

$$\left(\left[\theta_d(s)-\theta_o(s)\right]\frac{K_p}{K_v} + s\,\theta_d(s) - s\,\theta_o(s)\right)\frac{K_v}{Js^2+Bs} = \theta_o(s)$$

$$\left(\theta_d(s)\left[\frac{K_p}{K_v}+s\right] - \theta_o(s)\left[\frac{K_p}{K_v}+s\right]\right)\frac{K_v}{Js^2+Bs} = \theta_o(s)$$

$$\theta_d(s)\frac{[K_p+K_vs]}{Js^2+Bs} - \theta_o(s)\frac{[K_p+K_vs]}{Js^2+Bs} = \theta_o(s)$$

$$\theta_d(s)\frac{[K_p+K_vs]}{Js^2+Bs} = \theta_o(s)\left[\frac{K_p+K_vs}{Js^2+Bs}+1\right]$$

$$\frac{\theta_o(s)}{\theta_d(s)} = \frac{K_p+K_vs}{Js^2+Bs+K_vs+K_p} = \frac{K_p+K_vs}{Js^2+(B+K_v)s+K_p} = H(s)$$

$$= \frac{\frac{K_p+K_vs}{J}}{s^2+\frac{(B+K_v)}{J}s+\frac{K_p}{J}} = \frac{N(s)}{D(s)}$$

(ii) $D(s) = s^2 + \frac{(B+K_v)}{J} s + \frac{K_p}{J} = s^2 + 2\zeta\omega_n s + \omega_n^2$

Find K_v in terms of other terms for critical damping

↳ denominator of transfer function → for ~~overdamped~~ solution $\zeta \geq 1$

(when $\zeta = 1$ critically damped)

$$\omega_n^2 = \frac{K_p}{J} \qquad \frac{B+K_v}{J} = 2\zeta\omega_n$$

$$\omega_n = \sqrt{\frac{K_p}{J}} \qquad \frac{B+K_v}{J} = 2 \times 1 \times \sqrt{\frac{K_p}{J}}$$

$$\boxed{K_v = 2\sqrt{K_p J} - B}$$

(iii) K_p for steady-state error of $e_{ss} = 10^\circ$ for $\theta_d(s) = \frac{1}{s^2}$ and $B = 1$

$$e(s) = \theta_d(s) - \theta_o(s) = \theta_d(s) - H(s)\theta_d(s) = \theta_d(s)\left[1 - H(s)\right]$$

$$e(s) = \theta_d(s)\left[1 - \frac{K_p + K_v s}{Js^2 + (B+K_v)s + K_p}\right] = \frac{Js^2 + (B+K_v)s + K_p - [K_p + K_v s]}{Js^2 + (B+K_v)s + K_p}$$

$$e(s) = \frac{Js^2 + Bs}{Js^2 + (B+K_v)s + K_p}\theta_d(s)$$

$\theta_d(s) = \frac{1}{s^2}$

$$\lim_{t \to \infty} e_{ss} = \lim_{s \to 0} s\, e(s) = \lim_{s \to 0} s\left[\frac{Js^2 + Bs}{Js^2 + (B+K_v)s + K_p}\right]\theta_d(s)$$

$$= \lim_{s \to 0} \frac{s}{s^2}\left[\frac{Js^2 + Bs}{Js^2 + (B+K_v)s + K_p}\right] = \lim_{s \to 0}\left[\frac{Js + B}{Js^2 + (B+K_v)s + K_p}\right]$$

(iii) continue

$B = 1$ $e_{ss} = 10^0$

(e) $\lim_{t \to \infty} e_{ss} = \dfrac{B}{K_p} = 10 = \dfrac{1}{K_p}$ $K_p = \dfrac{1}{10}$